

# Asymptotic estimations of power thresholds and anti-Stokes frequency of laser induced thermal scattering in microresonators

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A derivation of the asymptotic expressions for the threshold intensity of laser induced thermal scattering in silica microresonator when illuminated with a plane wave is present. The calculation of anti-Stokes thermal combination frequencies are made for the spherical high Q-factor microresonators. The three modes regime of nonlinear interaction is considered. One pump-driven, two signal modes, and one mode of temperature relaxation are taken into account, satisfying morphology-dependent input and output resonances. There are low power thresholds for laser induced thermal scattering at morphology dependent resonances. © 2008 Optical Society of America

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Stimulated thermal Rayleigh and thermal Mandelshtam-Brillouin scattering from bulk liquids have recently been reported [1–3, 5]. The observations have been treated theoretically in terms of entropy and localized thermal fluctuations [4]. Rapid thermalization of the molecules which absorb at the laser pump, and the subsequent generation of large density or temperature fluctuation, leads to the enhancement of the thermal density fluctuations and increase the anti-Stokes frequency shift in bulk. There are numerous thermal nonlinear effects in silica microresonators due to the extremely high Q-factor of electromagnetic modes coinciding with "Morphology Dependent Resonance" conditions (MDR) [6, 16, 17]. At the specific threshold intensity of the laser pump and at the resonant size parameter of the cavity, the surface layer of the cavity significantly enhances the internal field inside the cavity at the incident resonant wavelength of laser pump, and further more efficiently provides optical thermal feedback from the internally generated

wave through optical bistability and instability effects [12–15]. The time dependent shift of the eigenfrequencies of the "Whispering Gallery Modes" (WGM's) can be caused by intensity-dependent mechanisms, such as the change in the refractive index which is dependent on laser light intensity and caused by an absorption-induced temperature change. The time dependent shift of the eigenmode frequencies is due to heating by laser power absorption, creating periodic thermal oscillations (oscillation regime) in spherical high Q-factor silica resonators [12, 13] and aperiodical oscillation (bistability regime) of scattering amplitudes, which have been experimentally investigated in microtoroid resonators [14, 15]. These experimental observations of the threshold oscillations and gain enhancements are associated with thermal nonlinear MDR and nonlinear interaction of WGM's in microspheres. The purpose of this communication is to report the calculation of thresholds of a new stimulated effect, which is believed to be attributable to laser induced stimulated thermal scattering in microresonators. The effects of the laser induced thermal scattering (LITS) in microresonators at MDR conditions has not been discussed in the literature. This article reports new knowledge relating to LITS, applicable to microresonators. The dielectric permeability of the resonator substance depends on the temperature of the resonator and energy in the volume of interacting modes which absorbed the laser light power. The dielectric permeability of the microresonator determines the relative shifts of the eigenmodes of the mi-

croresonator. A change in the dielectric permeability leads to a shift in the eigenmodes excited by the laser pump in the microresonator. Furthermore, independently, due to the heating of the resonator by the laser pump in input MDR conditions, the eigenfrequencies of the microresonator also have a periodic thermal shift. The maximum absorption of the laser power and heating occurs at MDR frequencies and a positive shift of the eigenfrequencies occurs when heat is released from the surface of the resonator. Alternatively, when heat is generated at the surface of a resonator, the eigenfrequencies of the resonator take on a negative shift. The thermal coupling of the electromagnetic modes in a microresonator provides the appearance in the scattering spectra of additional anti-Stokes thermal frequency shifts. One would expect a lower threshold intensity of LITS to be caused by thermal mode overlapping and MDR conditions simultaneous with Raman lasing. The lowering of the threshold of the Raman laser emission in silica resonators (with radius  $R = 40 \mu m$ ) has been reported [7]. Early theoretical explanations and the estimations of the low power thresholds for Raman lasing in microspheres have also been reported [8, 9]. Three modes regime of interaction were considered. The first mode is the thermal mode, the second is the signal and the third is the pump mode. Maxwell's electromagnetic equations and thermal equations for one thermal mode were solved by applying the methods of slow varied amplitudes for the system of ordinary differential oscillation equations [10]. If we take into account the

partial wave amplitudes of the thermal oscillations, and compute the scattering power of a resonator, (introducing Q-factor as the ratio of the field energy inside the mode to incident power, and multiplying by the leakage rate for power threshold) then we set [11]:

$$P_{th} = \frac{\omega_p^2}{2\pi Q_i Q_j K_{ijk} H_{ijk}} \frac{k}{\rho C_p} \left(\frac{\mu_i}{R}\right)^2 \quad (1)$$

Here:  $K_{ijk}$  and  $H_{ijk}$  are the thermal and electromagnetic mode overlapping coefficients estimated below,  $i$  corresponds to the thermal mode,  $j$  and  $k$  correspond to the electromagnetic eigenmodes;  $\omega_p$  is the pump frequency,  $Q_i$  and  $Q_j$  are the Q-factors of  $i$  and  $j$  eigenmodes,  $\mu_i$  is the eigenvalue of thermal conductivity equation [11],  $\rho$  is the density of the resonator,  $k$  is the thermal conductivity,  $C_p$  is the specific heat capacity,  $R$  is the radius of resonator. The threshold power is obtained at MDR conditions for the optimal tuning conditions:  $2(\tau_t + \tau_e) = 1 + \omega_f^2/\omega_p^2$ , where  $\tau_t = k(\mu_i/R)^2/\rho C_p \omega_p$  and  $\tau_e = 1/2Q_i$  are the relaxation times of thermal and electrical modes with  $\omega_f$ . Electromagnetic decay rate of the effective three-mode interaction is  $\tau_e$  and the thermal relaxation rate is  $\tau_t$ . The frequency  $\omega_f$  can be calculated by the transcendental eigenvalue equations of Mie's theory using the resonant size parameter at MDR. For the optimal tuning conditions and the lowering of the LITS threshold by MDR conditions, the thermal anti-Stokes frequency is [11]:

$$\Omega_T = D \left(\frac{\mu_i}{R}\right)^2 \quad (2)$$

Here:  $D = k/\rho C_p$ . Although two high Q-factor circuits are present in the microres-

onator, the optimal detuning during three-mode LITS can be small if the frequency interval between the resonator modes is approximately equal to  $\Omega_T$ . WGM's efficiently provide the thermal feedback for internally excited modes at the specific frequency matching:  $\omega_p = \omega_s \pm \Omega_T$ . Equations (1),(2) provide the information concerning the absorbed substance of a resonator as well as the yield of the multi-exponential thermal damping of any stimulated processes at times comparable with a thermal oscillation in Eq.(2). The mode overlapping coefficients  $K_{ijk}$  and  $H_{ijk}$  have a complicated structure, and consist of the high oscillating eigenfunctions [11]. The mode overlapping coefficients have the following form [10]:  $H \simeq \omega_f^2 a_\epsilon T$ ,  $T^2 \simeq (\rho C_p V)^{-1}$  and  $K \simeq \sigma T$  where:  $\sigma = \epsilon \omega_f / 4\pi Q_i$ ,  $\epsilon$  is the dielectric permeability,  $a_\epsilon$  is the temperature coefficient of dielectric permeability,  $V$  is the effective volume of interaction of WGM's [16]. The calculations were made by using the known values of material parameters valid for the experiments: density of fused silica  $\rho = 2.21$  [g/cm<sup>3</sup>], thermal conductivity  $k = 1.4 \cdot 10^{-2}$  [W/cmK], specific heat capacity  $C_p = 0.67$  [Ws/gK],  $a_\epsilon = 1.45 \cdot 10^{-5}$  [K<sup>-1</sup>],  $D = 9.5 \cdot 10^{-3}$  [cm<sup>2</sup>/s], refraction index  $n = 1.46$ ,  $Q_i = 10^8$  [17]. An asymptotic estimate of the mode overlapping integral is valid for a high index of partial wave amplitudes of WGM's  $n \simeq x$  (here  $x$  is the resonant size parameter) for  $n$  surface modes obtained from Cauchy-Bunyakowski-Schwarz inequality [10]. The mode overlapping coefficients  $K$  and  $H$  were written under the following conditions: (1) small losses of WGM's, which

are appropriate for the high Q-factor of the resonator modes; (2) singling out two interacting modes within the homogeneous thermal mode volume frequency interval. One is the pump mode and the other is the resonant signal (anti-Stokes) mode, namely, both input and output resonance conditions [6]. The input resonance condition is satisfied for a broadband thermal detuning of the input and output modes, which spans several high-Q MDR's, whereas the output resonance condition is always satisfied, since the bandwidth of LITS spans at least several high-Q MDR's. Substituting the asymptotic estimation for the mode overlapping coefficients  $K$  and  $H$  in Eq.(1) for the threshold power of LITH yields:

$$P_{th} = \frac{2kV}{\varepsilon a_\varepsilon Q_i} \left( \frac{\mu_j}{R} \right)^2 \quad (3)$$

The threshold of LITS is then determined by two circumstances: the effective resonant heating absorption and the overlapping mode coefficients [11]. In Fig.1 the dependence of the threshold power on the fused silica spherical resonator radii is shown. The interacting nondegenerate WGM's "TE", ( $TE_n, n \simeq x$ ), ( $x$ - resonant size parameter) are coupled to thermal modes by thermal nonlinearity at wavelength of 840 nm. The computed threshold input intensities in a silica nonlinear sphere vary from 2 to 23  $\mu W$  for radii of 40 to 110  $\mu m$ , the Q-factor of a "TE" resonant mode is  $Q_i = 10^8$  [17]. As illustrated in Fig.2, the threshold incident pump power  $P_{th}$  of LITS depends on the thermal  $T_n^1$  mode order. More quantitatively the threshold pump power for LITS Eq.(3) is found to

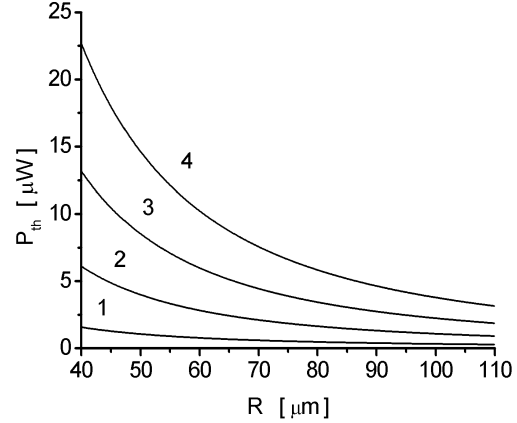


Fig. 1. The threshold power of STS for fused silica microsphere. Radius and thermal mode dependence: 1.  $TE - T_1^1$  modes, 2.  $TE - T_3^1$  modes, 3.  $TE - T_5^1$  modes, 4.  $TE - T_7^1$  modes. The laser pump is mode-locked  $Ti : Al_2O_3$  laser operating at 840 nm with the pulse repetition frequency 82 MHz and the pulse duration 1 ps.

be less 50  $\mu W$ ,  $1 \leq n \leq 8$ , for the radius of 40  $\mu m$ , well below the threshold of Raman lasing 86  $\mu W$  [7] and stimulated Brillouin scattering in glass sphere, which has the order of 160 W [18]. To be specific, the threshold power of LITS is less than the threshold of stimulated Raman lasing and stimulated Brillouin scattering in silica microspheres, thus the thermal interaction of the eigenmodes can be revealed by the spectra of the Raman lasing with thermal instability or bistability in microresonators. As illustrated in Fig.3, the thermal anti-Stokes combination frequencies are dependent on the number of the interacting modes. The thermal anti-Stokes combinational frequency of LITS can be varied over a wide range of frequencies

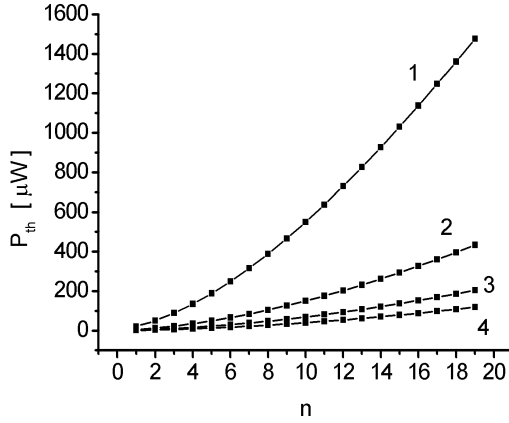


Fig. 2. The threshold power of LITS of  $T_n$  modes with radius of sphere  $R = 40 \mu m$  1.  $TE - T_n^1$ , 2.  $TE - T_n^2$ , 3.  $TE - T_n^3$ , 4.  $TE - T_n^4$ . The pump is at  $1.55 \mu m$  of external CW-diode laser with  $300 kHz$  line-width [7].

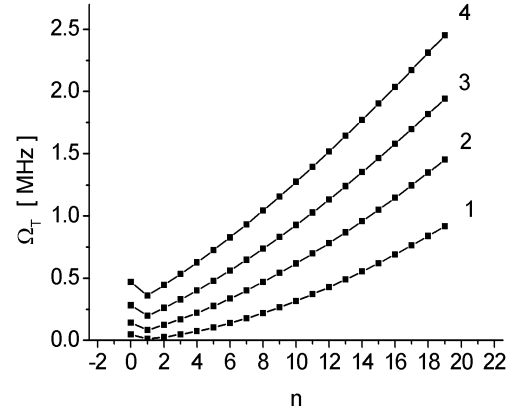


Fig. 3. The Anti-Stokes combination frequency of LITS from fused silica microsphere irradiated by CW-diode laser pump at  $1.55 \mu m$  [7],  $R = 40 \mu m$  1.  $TE - T_n^4$ , 2.  $TE - T_n^3$ , 3.  $TE - T_n^2$ , 4.  $TE - T_n^1$ .

from Rayleigh's frequency of shift via thermal Rayleigh scattering [3] to the striction combination frequency of thermal Mandelshtam-Brillouin scattering [2,4]. Such a wide range of anti-Stokes frequency can be provided by systems such as a spherical resonator. The anti-Stokes frequencies of LITS can occur in spectra of Mandelshtam-Brillouin, Raman scattering and linear Mie's scattering. The frequency can be varied over the wide range and depends on the material parameters of the resonator as well as both the effective volume of WGM and the surface mode overlapping. In order to have the description of Raman scattering and lasing in agreement with the experiments it would be necessary to take into account the thermal modes in the set of electromagnetic equations. The difference in the threshold power of interacting thermal modes enables the separation of these nonlin-

ear processes from each other. The thermal oscillations and the thermal spatial overlapping of electromagnetic modes are important processes for Raman scattering and lasing in microresonators.

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